The wealth repartition law in an altruistic society.

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Abstract.

Econophysics and economy simulations claim to have demonstrated that the repartition of wealth in every economic systems always occur in a very inegalitarian mathematical law, called the Pareto law. As this inegalitarian repartition is a cause of suffering and injustices in the real world, it would be interesting to find a remedy. We can note that the econophysic studies always start from the hypothesis as what economy systems are formed only of agents perfectly egocentric, each seeking only to gather the maximum of wealth for himself. Should the Pareto law come only of this limiting hypothesis? And if agents had other types of behaviours, for instance altruistic? This study is based on a very simple simulation where we can program various rates of altruism and egocentrism. Really we can check that a much more egalitarian repartition appears, even with a relatively low rate of altruism (15%). More, this egalitarian repartition occurs according to a completely different law of that of Pareto: a Gauss law, a bell curve.

Key words: econophysics, inequalities, Pareto, altruism, collaboration, general Epistemology, True Economy.

Econophysics and the topic of altruism

The **basic principle of econophysics** is to study the evolution of an economy system using an **analogy with physical systems,** while comparing the economy agents (individuals, enterprises) to simple physical objects (molecules, particles) which behave all in the same way. Starting from there, it is possible to conduct statistical analysis and deduce economical laws, as we do with physical models. So the econophysicists are able to make checked predictions, for instance on the repartition of income or on stock market fluctuations. See some basic presentations such as the ones of Mantegna and Stanley [Ref. 1], or De Liso and Filatrella [Ref. 2]

For the econophysicists, the ideal economy agent behaves in an exclusively **egocentric** way. And he will use **strategies** to pursue his egoistic interest and try to bring to him the maximum possible wealth. The econophysicists even call this «to behave rationally». This type of behaviour is the recognized basis of a real economy system such as the capitalism.

The most well-known econophysics prediction is the **Pareto law,** from the name of its discoverer, the sociologist and economist Pareto, at the beginning of the 20th century. After this law, the incomes always show a decreasing exponential looking repartition (the exact name depending on the mathematical formula used, power law, lognormal, of Levy…) between a small minority of rich, a minority middle class, and a large majority of poor peoples.

Such a law, which poses the social and economy inequalities as something mathematically unavoidable, may be of good comfort for the small minority of rich, but it is a tragedy for the billions of persons who suffer in their human dignity, see in their flesh. This feeling of injustice is thus the main cause of numerous problems, ranking from benign social unrest to international terrorism. So thinkers call for a remedy, for instance while proposing a more altruistic behaviour. And really we enjoy this possibility to behave in an altruistic way.

Altruism is a constant trend in life history. About the animal realm, ethologists and sociologists study altruism since long ago (at least as a behaviour), under the name of collaboration [Pepper and Smuts, ref 3, or Gintis, ref 4] or reciprocity [Danielson, ref 5]. Forms of altruism were common among human tribal societies. All the religions advocate altruism and sharing, and numerous religious or lay communities attempted to concretise such visions, from evangelical times until today. In the Middle Age in Europe existed monasteries gathering sometimes thousands of persons, running on the basis of a selfless activity of the individual for the good of the group. In these troubled times, their vast domains were oasis of abundance and peace for the peasants who lived in. The same principles were systematically put at work in the Buddhist world, ancient or modern: The whole set of Buddhist communities and art works exists only through benefactors. The modern world sees the appearance of the humanitarian action and fair trade, and both, in some years, achieved major roles in certain precise domains.

Can really altruism lead to a society where wealth is shared in a fair way, or does the Pareto law condemns us all to inequality even in this case? The purpose of this study is to try to understand what happens when, precisely, the economy agents behave **in an altruistic way**.

Mathematics of the Pareto law.

It was proposed several repartition laws, but the most common is that of Pareto, or the power law. Here is a very good small tutorial: «Zipf, Power-laws, and Pareto, a ranking tutorial» by Lada A. Adamic, [ref 6].

The Pareto law gives the repartition of fortunes P larger than p according to the law:

$$(1)\ P = x^{-k}$$

with k comprised between 1 and 2

The power law gives the repartition of fortunes p according to the law:

$$(2)\ p = x^{-a}$$

with a = k+1.

Example: if in a system k=1.5, then a=2.5.

We note that the first formula is the integral of the second (it is a cumulative distribution). So the two laws describe the same reality, and the following graphs will be power laws (2).

A system where k is low is rather inegalitarian, but if k is high, the system is more fair. Real economy systems as well as simulations give k values ranging from 0.5 to 2, even in a single country [Taladidia Thiombiano, ref 7]. A low value of k gives a very inegalitarian society. Larger values of k give less inequality, but are still far from a real equity. Other laws for the repartition of wealth were proposed, for instance exponential [ref 8]. But this does not change the overall look of the graph, nor its practical humanitarian consequences.

On a graph, the eye does not easily distinguish a power curve from another curve, and it is unable to estimate its power. So we shall use on display 3 of the simulation a peculiar presentation called log-log [ref 6] where we display the logarithms of the abscissas and of the ordinates. With this display, also called asymptotic diagram, a power law curve appears as a straight line, which slope is proportional to the power coefficient. So a power law is easily checked with the eye, and it is easy to measure its power and to perform mathematical tests such as linear regressions.

At last we shall note that the Pareto law, as such, makes no physical sense: in any system, there would be an infinite number of persons with a zero income! In reality the formula is valid only in a certain interval. Before and beyond this interval, there are still some values, but which do not obey the overall law, and which make «tails» which can involve 20% of the whole number of agents.

A software simulation to point at the role of altruism

For this demonstration I created a **simple economy simulation,** which puts on stage a small society of all identical agents, who will try to get richer while investing the one on the other.

My first idea was to put this simulation on line, to make it available for everybody. For this reason, I wrote it in JavaScript, a programming language which can run on a standard Internet page without any downloading. But, unexpectedly discovering the Gaussian law, I thought that this result had a scientific value and that it deserved to be first published according to recognized science standards.

First I was looking only for a simple and evident visual result, for peoples without scientific knowledge. But I had to add various functions allowing giving the necessary scientific justifications. So the simulation features several displays (bar graphs) and fields for numerical results, to be able to gather them in a spreadsheet. Input fields allow to adjust certain parameters and to introduce altruism in a gradated way, and with several methods.

This simulation is very simple, as it contains only a number NU of economy agents (two hundred) all identical, which only interaction is to invest the one on the other at each turn. The underlined words are name of main program parameters.

At the beginning of the game, we can add a start quantity of money for each agent. Then, at each turn, we first add a constant income for each agent, before calculating the interactions. To avoid a systematic bias for certain agents according to their row, at each turn a number NU of rows *m* are drawn at random to select the agents who will play. So the agents play in a random order, different at each turn. Then, for each agent *m*, we draw a second number *n*, on whom the agent m will interact. Then for each agent pair *m* and *n*, the egocentric and altruistic actions are evaluated and done, according to the values of the parameters investment, altruists and altruism.

In the case of an egocentric action, the agent *m* invests a proportion investment of his fortune, which is his investment. A random drawing gives the result of the interaction. The agent *m* can lose up to half of his investment, to the advantage of *n*, but he can also gain up to twice, to the disadvantage de *n*. If the fortune of *n* is lower than that of *m*, to avoid the appearance of negative values, so the interaction is calculated on the basis of the fortune of *n*. This is realistic, as a rich shopkeeper cannot earn much money from a poor customer.

If we program a percentage of altruism, this proportion of the investment will on the contrary be shared in favour of *n*. (If *n* is richer, the action does not take place.). If we program a proportion altruists of agents, so these agents will only perform an altruistic action with their whole investment. They appear as red bars.

At last, if we give a non-null value to the Socialism parameter, a function takes a toll of the fraction Socialism on the 5% largest fortunes, and shares the collected sum to the 20% poorest.

The graphs (displays) all have an automatic scale, and this can sometimes give a feeling as what the weaker fortunes are lowering, even if in fact they grow.

Default values and some secondary parameters can be easily modified at the beginning of the JavaScript, with a simple text processor. Search for «INITIALISATION OF KEY VARIABLES».

The JavaScript language allows everybody to run this software, on an ordinary Internet page. But more detailed and more realistic simulations (with several kinds of agents, flux of merchandises, regions, etc.) are beyond the possibilities of the JavaScript, a slow language with only limited functions.

The effect of altruism in the simulation

When we play some turns of the simulation, with all kinds of altruism equal to zero, we really observe that it quickly appears a small minority of ultra-rich, a minority middle class, and a large majority of poor agents, according to a classical power law. This law appears under the form of an exponential looking curve on the Gaussian display 4, or as a straight slope on display 3 log-log.

What this simulation brings new is that we can also study what happens when the economy agents **behave in an altruistic way.** We can **introduce altruism,** gradually and in three different manners: a Socialism tax on fortune, a percentage of peoples being totally altruists, and a proportion of altruism identical among all the agents. When an altruistic agent interacts with another, he shares in place of investing to exploit. The proportion on investment or of sharing is that given by the altruism parameter.

We can note that weak values of altruism change nothing to the power law. (Protocol: default values, Gauss display, play 100 turns between each modification of altruism) The displays 4 and 5 both show an exponential-looking curve (trumpet shaped). But after a certain threshold of altruism, as low as 12%, the repartition of the fortunes changes drastically. In a range of only 3-4%, the curve completely changes shape. **We no longer have a power law, but a repartition of a completely different type: a Gaussian repartition, a bell curve** (visible with the Gaussian display 5) While still increasing altruism, **the inequalities swiftly get lower:** from one to two at 30%, negligible at 50%. Let us note that **Fair trade** may correspond in this simulation to an altruism rate of at least 13%.

Introducing a **form of state ruled altruism** socialism gives similar results, but less efficient. (Protocol: default values, Gauss display, play 10 turns between each modification of socialism) We pass from the power law to the Gaussian law in an interval of about 10 to 20%, and the inequalities are in a two proportion at 50%. Above, the spoliation of the riches introduces a new form of inequality! A real system has to be more gradated.

Whatever the proportion of 100% altruists agents in an egocentric society, they always keep themselves within a fortune level much lower than the 100% egocentric agents. (Protocol: default values. Play 100 turns between each modification of altruists. Use the log or gauss display). If the proportion of altruists agents is low, they are even excluded of the economy play. This situation is very likely the one in western capitalist societies, where altruistic behaviours are a minority. They have little impact on the running of the whole system, and this explains that they escaped the attention of econophysicists. However we note that the altruistic agents, among themselves, manage very homogenous fortune levels, with low inequalities. And **a high rate of completely altruist agents are able to maintain a very correct living standard,** but without avoiding the existence of a class of rich. This situation is what happens in certain Third World countries, where a real solidarity in the people avoids the complete crushing of the poorest. But these countries cannot protect themselves of rich foreign companies nor from local oligarchies.

At last, if we **combine various forms of altruism,** their effects reinforce each other. For instance a society with a higher rate of altruists agents (60%) needs only 20% of socialism to maintain all the fortunes within a 1 against 2 range. We think to the Third World countries already noted: if they could tax the foreign companies, even moderately, they would quickly achieve a decent living standard for all. At last, in combining the three forms of altruism, a one against two repartition appears with only 15% of each.

So, with few altruism, we have a strongly inequalitarian economy system. With enough altruism, this system becomes equalitarian. We even not need to be all saints: in the simulation only 50% of altruism in enough to lower inequalities to less than 20%. So to get the today observed results in economy really needs a systematic chase against every form of altruism!

Detailed discussion on the validity of the results.

Who becomes rich? The first display mode (Linear, display 1) allows to check that the agents who become rich are never the same for a game to another. So there is no objective reason which favours some. If the simulation starts of the hypothesis of the agents being all strictly identical, it is really to show this fact: The inequalities do not come from the various capacities of the agents, but they are a consequence of the intrinsic instability of the system.

But we also find, after only some turns, a well known fact: rich tend to stay rich, and poor people tend to stay poor.

The power law. The first check to perform on the simulation was about it really obeying the Pareto law. Otherwise could have appeared the argument that the following results cannot apply to systems obeying the Pareto law.

The power law first appears from the shape of the curves, when we play with all the forms of altruism at zero. The figure 1 (fortune per agent) clearly shows an exponential looking law, perhaps a power law, at least for the largest fortunes.

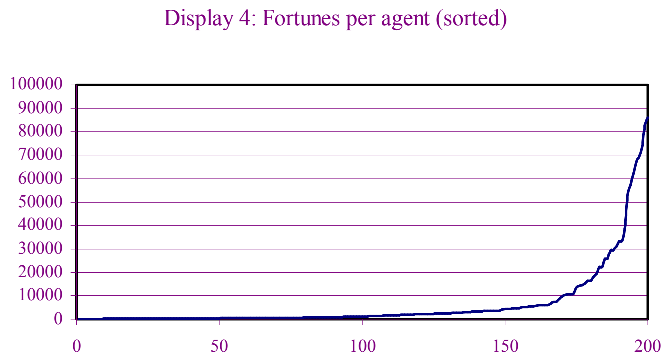

Figure 1

This display is certainly visually appealing, but it shows only a part of the results, and it does not indicate the exact formula, for instance its power. For this reason, it is better to use the display 3, log-log, which shows a power law under the form of a straight line. A problem is that this display is not accurate, as statistical fluctuations hide the expected shape. So we must make an averaging, with the addition of the results in a spread sheet.

Protocol: With default values, we play 1000 turns and copy the field 2 ordinates in a line of a spreadsheet. We then play an hundred turns and copy the new results under the previous, 20 times. At last we copy the field 3 of abscissas in the spreadsheet. We calculate ordinates with the average of results per abscissas. Before creating the graph, we must note that, for the abscissas are already in a logarithmic scale, the abscissas does not come with equal intervals, so each ordinate must be divided by the interval between its abscissas and the next abscissas, in order to obtain the real plot of agents per fortunes. At last we create the graph, with a logarithmic scale for the ordinates. The result is shown in the figure 3 under.

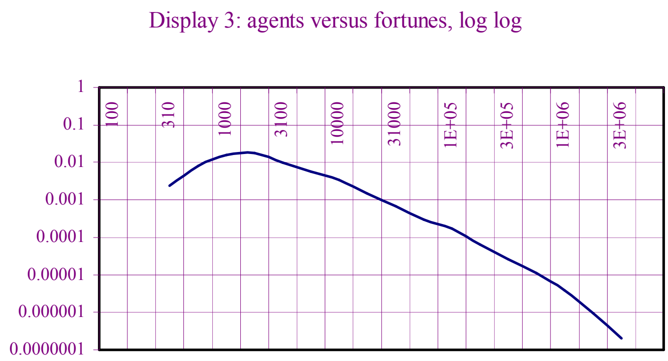

Figure 2

This curve strongly resembles what we expected from a power law ranging between two limits: a section of straight line with two tails or undefined zones at the extremities. We can estimate its power from two points, with an accuracy matching correctly that of other econophysics studies. Fortunes 100 000/agents 0.0001, and fortune 10 000/agents 0.003. (A decade of ordinates is with four spaces). That gives a power of -1.5, and a Pareto coefficient of -0.5. Such a low power gives very strong inequalities, to be compared to powers generally observed, rather in the -0.5 -2 range.

So our system really obeys the Pareto law, and in an extreme way. This will be interesting later, if altruism is able to correct such a savagely inequalitarian system.

The Gaussian repartition appears as soon as altruism reaches 15%. It is clearly visible on the graphs. On display 2 (log of the fortune, per agent) it shows as a horizontal line. On display 4 (Fortune per agent, sorted) it appears as a leaned S curve. At last on display 5 (frequencies of the fortunes) it appears as a classical bell curve. Figure 3 shows an example obtained after averaging of numerous results.

Protocol: With the default values and Gaussian display, set altruism at 35%, play 100 turns, and copy the field 2 ordinates in a spreadsheet. Restart the game and redo 20 times the previous, checking each time that the abscissa scale did not changed. Copy the field 3 abscissas in the spreadsheet. In the spreadsheet, average the ordinates for each abscissa and generate the graph.

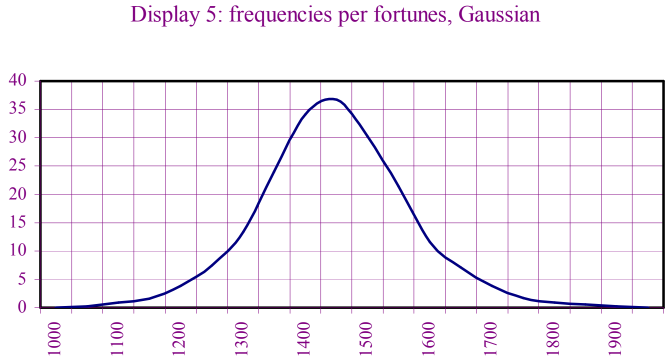

Figure 3

This curve looks quite Gaussian. But, as this statement is the main assertion in this study, a visual check was not enough. It deserved a more accurate check, with a thorough analysis of numerical data. For this I created some more lines in the spreadsheet: normalisation of the results to 1, Neperian log, square root, and changing of sign for the relevant half, in order to obtain the linear function f(x) which is in the Gauss formula: $y = e^{-(f(x))^2}$ where f(x) is a linear function of the abscissa x. This gives the graph on figure 4:

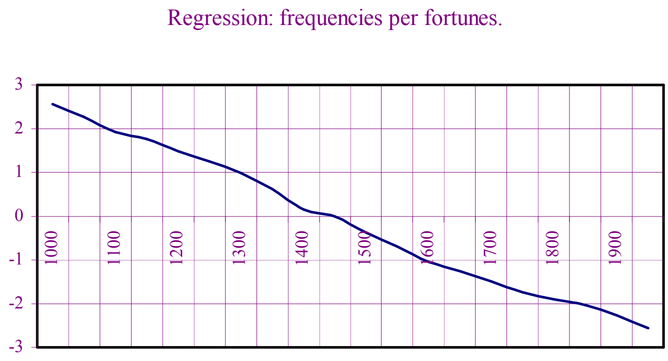

Figure 4

At last a linear regression test on this curve gives a value of 0.9936, that will confirms that it closely matches a straight line, and thus that the previous curve is really close to a Gauss curve. We can note that to set altruism at 35% is not at random. I tried with other values, but with too low values, we get closer of the zone where the Pareto law becomes stronger, and the

Gauss curve becomes asymmetrical. With greater values, the Gauss curve becomes very narrow, and the checking is more difficult. But it is still a Gauss curve.

So we can safely conclude of this study that, when agents behave in a more altruistic way, the distribution of the wealth completely escapes from the Pareto law and obey to a Gauss law.

The effect of individual differences. The purpose of taking all identical agents was to show that the inegalitarian repartition of Pareto is not an effect of any supposed superior merit of certain individuals, but only of statistical hazards. But in the simulation, high levels of altruism lead to a complete levelling of the fortunes. It is clear that in this case the hypothesis as what all the agents are identical is no longer realistic. We can test this point while introducing some difference, with the altruists parameter, which make a percentage of the population completely altruistic. We can play with the default values, Socialism at zero, altruits at 40%, and altruism at 18%. This test is shown on figure 5.

Protocol: Play 1000 turns. Play ten turns and copy the field 2 ordinates in a spreadsheet. Redo ten times the previous, while checking that the abscissas values did not changed (if that happens, restart the game). Copy the field 3 abscissas in the spreadsheet. In the spreadsheet do an average of ordinates for each abscissa, and then create the graph).

Display 5: Test of individual differences

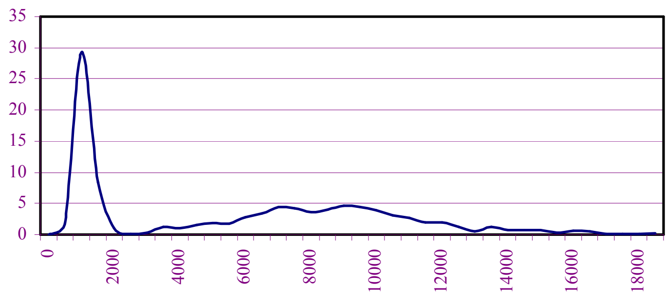

Figure 5

Figure 5 was obtained with altruists=40 and altruism=18. It clearly shows a bimodal repartition. The comparison with display 4 indicates that the sharp bump at the left is the gathering of completely altruistic agents, while the limp bump in the middle is the gathering of relatively egocentric agents. Test with higher levels of altruism still show two bumps, but they get closer and even get confused with altruism higher than 27%.

Classical scientific conclusions.

The main conclusion of this study is that **we can state that the law of the repartition of wealth in an altruistic society can differ of the Pareto law.** It can be for instance a Gaussian law. And then that this repartition is much more fair than in a society of egocentric agents ruled by the Pareto law. This is true even in this example however extreme (power 0.5).

Not only the altruistic economy can be much more efficient that the egocentric one, but in more **the egocentric economy and the altruistic economy would run in a completely different way.** Really, in this simulation, when we lower altruism from 30% to 15% we observe more and more unsymmetrical and spread Gauss curves, which end to more and more resemble Pareto curves. To compare things with physics, we can say that we have a phase transition between a Gauss economy and a Pareto economy, as between solid and gaseous state. Egocentricity plays the role of temperature to pass from an ordered (Gaussian) state like solid, to a disordered (Pareto) state like gas.

We can even state that **the Pareto law, far of being universal and unavoidable, would be only the peculiar way of functioning of an egocentric society**. There is even a hint of a much more general mathematical law, which passes progressively from a Pareto law to a Gauss law.

We also noted that the differences between the agents do not come from the various capacities of these agents, but they are a consequence of the intrinsic instability of the system. This result is anyway already known by econophysicists, even if the politicians did not draw the conclusions: «We show that the Pareto wealth distribution is a robust consequence of a fundamental property of the capital investment process» [Moshe Levy, ref. 9].

If, as state the econophysists, the discovery of the Pareto law is a science result, so the discovery of the Gaussian distribution is as much a science result. The appearance of this law shows that a different functioning is perfectly possible, without generating social inequalities.

The only restriction to this study, but it is a large one, is that we need to find other examples of systems obeying the Gauss law, maybe simulations, maybe real societies, for instance communities with strong religious commitments. I think to certain Indian communities un USA or Canada, some muslim closed towns in south Atlas (Algeria or Morocco) ore some communities in Himalayas. If such studies also show a Gaussian repartition, **so we shall be able to state that the Gaussian repartition characterizes an altruistic economy, when a Pareto repartition characterizes an economy based on egocentricity.**

A secondary but noticeable result is that figure 5 contradicts one of the basic tenets of Marxism. This tenet states that the mind and behaviour of peoples depend on their social class. Poor class workers would be altruistic toward their comrades, and rich class owners would have a «bourgeois» selfish mind. Two statement which led respectively to populism and to persecutions. But if this tenet was true, we should observe a bimodal repartition of wealth like in figure 5. A bimodal repartition which precisely was never mentioned by econophysicists: they always observed Pareto curves of various power, but continuous (monotonous). So this makes clear that, as psychology and spirituality predict, our mind and behaviour do not really depend on our social position, but on more personal conditions. This also show that everybody, poor or rich, is equally responsible for the working of the society and for the overall wealth repartition.

These results show that there is no mathematical forbidding to an equalitarian distribution. On the contrary, while choosing to behave in an altruistic or egocentric way, we could choose to place ourselves in an equalitarian or inequalitarian system. So the only cause of inequalities is the egocentric behaviour of all the agents, the poorest included.

While playing with the simulation we can observe that important inequalities appear from the first turn, despites the fact that the agents are all equal in income as well as in efficiency. So it is quite clear that the inequalities that we observe in reality are absolutely not the consequences of any kind of **personal merit or weakness.** The very purpose of taking all identical agents was to show that the inegalitarian repartition of Pareto is not an effect of any supposed superior merit of certain individual, but only of statistical hazards. See what says Moshe Levy, [ref 9]: It is the stochastic consequences of concurrence which enrich some to the detriment of the majority, leading to the Pareto repartition. «We show that the Pareto wealth distribution is a robust consequence of a fundamental property of the capital investment process». Of course, in real systems, the differences among agents necessarily favours some; but this is not the real cause of the observed Pareto repartition nor of the incredible inequalities we observe in the world.

In the simulation, high levels of altruism lead to a complete levelling of the fortunes. In reality the individual variations in skills, motivation, use of resources, would introduce differences even if the income would be strictly equal for everybody. But in this case there are no longer injustices, so we can speak of differences in place of inequalities. So, even with individual differences, an altruistic economy system does not introduce social inequalities of its own.

At last the equality obtained in an altruistic society is not a levelling down: **the living standard obtained is really what allows the available wealth,** in the simulation the income attributed to each agent, in reality what the ecosystem can produce. There is no poverty in abundance.

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